**School**

**Name**

**Course**

**Allocation of Risk Capital**

Contents

[List of Figures 3](#_Toc71974927)

[Introduction 4](#_Toc71974928)

[Risk measures 5](#_Toc71974929)

[Definition Returns 5](#_Toc71974930)

[More than one asset at Risk measures 6](#_Toc71974931)

[Theory Game theory on risk allocation 7](#_Toc71974932)

[Allocation of the atomic players 8](#_Toc71974933)

[Alternatives of Allocation 11](#_Toc71974934)

[Activity based method 11](#_Toc71974935)

[Incremental method 12](#_Toc71974937)

[By Beta method 12](#_Toc71974938)

[Lorenz method 12](#_Toc71974939)

[MATLAB 13](#_Toc71974941)

[Matlab Rules of implementation of allocation in Matlab 16](#_Toc71974943)

[Matlab program 17](#_Toc71974944)

[Conclusion 19](#_Toc71974945)

[References 19](#_Toc71974946)

# List of Figures

[Figure 1 10](#_Toc71895810)

[Figure 2 11](#_Toc71895811)

[Figure 3 12](#_Toc71895812)

[Figure 4 15](#_Toc71895813)

[Figure 5 15](#_Toc71895814)

[Figure 6 16](#_Toc71895815)

[Figure 7 17](#_Toc71895816)

[Figure 8 17](#_Toc71895817)

[Figure 9 19](#_Toc71895818)

[Figure 10 20](#_Toc71895819)

# Introduction

This research is based on allocation of risk capital. First of all, what is "risk"? The definition of "risk" has many ambiguous and contradictory meanings, causing widespread confusion and leading to the use of very different methods for risk management in different areas.Generally, risk refers to the possibility that the selected type of activity will lead to undesirable consequences. Businesses or financial entities may present many types of risks.

 **Market risk**: Risk is related to changes in the value of market risk factors. Related market risks include interest rate risk, exchange rate risk, capital risk and raw material risk.

 **Credit risk**: The risk that the borrower will not be able to fulfill the repayment obligations without repaying the debt.

 **Liquidity risk**: You cannot trade assets or securities quickly enough to avoid the risk of loss.

 **Operational risk**: Risks related to personnel, systems, and processes; such as fraud (such as fraudsters), technological disruption, external events (such as earthquakes, terrorist attacks), political risks (sudden policy changes in specific countries/regions), etc. Identify and assess risks, then use resources to control and monitor harmful potential.After identifying the risks, risk management personnel will try to minimize the impact of negative events. Therefore, incorrect risk management can have serious consequences for companies and individuals. This work only focuses on market risk, that is, the risk of a decline in the value of the investment portfolio.Due to price fluctuations. Like other types of risks, market risk is difficult to measure because it is not observable, unlike price, profitability, portfolio value, etc. Generally, there is a clear correlation between risk and return on investment: the greater the risk, the greater the expected return. Risk management at any timeInvestors quantify potential investment losses, and then take appropriate measures based on investment objectives and risk tolerance. In order to deal with the uncertainty of the future portfolio value, the company usually has some very low-risk investments that can be used as a buffer to reduce the possibility of accidental losses. This pillow is called the company's venture capital. Therefore, "risk capital" refers to the amount of risk-free capital added to the company's assets to ensure this.The future value of the company is accepted by management, chief risk officer, regulator or others. From a financial point of view, since the return on this amount is very low, holding an amount with little risk is considered a burden and can be invested in products with higher returns.Portfolio, business area or other types of business areas. If these businesses are to operate independently, each business will need to maintain its own specific amount of venture capital to make up for the potential losses of that particular business. Being part of the same company provides the benefits of diversification.

# Risk measures

This section introduces the basics of risk management. By definition, it is impossible to predict future transactions or venture capital, so we need to predict future results. First, we need to specify the time frame, which is the future time in which we are interested in risk measurement. The line of sight between agents is very different. Trading time ranges from a few hours toThe age of the pension fund. However, companies are generally interested in measuring the risk of year-end financial results. Now that you have determined the time frame, you need to estimate the distribution of portfolio value at a specific point in the future. All the parts of the distribution are all focused on the middle part of the distribution, with the emphasis on probability.For risk management purposes, we are most interested in the swing and the left tail. Despite the hesitation, the real fear is the other side of the coin. Enough to prevent the company from continuing its normal business. Therefore, rating agencies and regulators will mainly focus on the negative extremes on the left. Returns here is unknown for the protential distribution of market prices which makes comparative risk a problem.Due to the density function of the rate of return, it is almost impossible to determine the exact distribution of the financial rate of return. Since financial risks cannot be directly measured and must be maintained, the task of measuring financial risks becomes more complicated. Risk is a hidden variable, which means it cannot be measured the same as the weight on the scale. It can be said that if the price fluctuates throughout the day, the risk may be high. When measuring risk, we are very interested in changes in value. The investment portfolio is measured by profitability.

# Definition Returns

Two types of advantages are commonly used in the literature: Let Pt be the portfolio value at time t.

Simple returns

j –period returns:

But for smaller expenditures and shorter time horizons, geometric expenditures and arithmetic expenditures are almost the same. Which method chooses the correct method? In financial models geometric models are used when they can move between multiple time periods. For example, if rt1, t and rt2, t1 are normally distributed, then rt. In the figure, t is the sum of two normals that are also normally distributed and have closed distributions. The two-period distribution of returns is the product of two normal random variables. For example, the calculation gain is analytically allocated two cycles in a closed manner. The advantage of combining assets into a portfolio is that if rt and wk are arithmetic operations in the first period, one way to measure return risk is to assume a specific distribution of returns, such as B.Ordinary or student For example, in finance, geometric gains are usually considered normal, which is the same. Assume that the future price distribution is normal. Acceptance is the key to keeping the famous Black-Scholes option pricing formula simple. Income is usually not normally distributed, because the actual income distribution usually has a thicker tail than the normal distribution, especially on the left. However, the assumption of normality simplifies the model and fulfills many expected functions.

# More than one asset at Risk measures

Having an investment portfolio combined with different assets you need to consider the structure of the relationship between different assets. Due to the structure of dependencies, it may be difficult to understand the dependencies between different assets. Correlation plays an important role in financial theory. Although it has obvious shortcomings. First, the correlation is only determined for random variables with finite variance.However, this property may cause problems. Second, correlation only measures linear correlation. Therefore, if the two random variables X and Y are independent, the correlation (X, Y) is zero. Non-linear correlation structure. In addition, Corr (X, Y) 0 does not necessarily mean that the relationship between X and Y that ultimately strictly increases the nonlinear transformation is not a correlation.constant. For strictly increasing function 1, t2 is expressed as Correlation (t1(X), t2(Y)) related (X, Y).

# Theory Game theory on risk allocation

We saw some of the several risk indicators and which it converted the risk into numbers. Now let us remember our multidisciplinary organization. By establishing standards for measuring risk, each business area can measure the risk of its own investment portfolio. However, if there are multiple departments. Establish alliances to integrate some of your investmentsInvestment portfolios in large investment portfolios can have multiple advantages, namely risks. The alliance can be less than the sum of independent risks. Remember, if the risk indicators are met, the risk of the large portfolio is not greater than the sum of the risks of the sub-combinations (sub-additivity requirements). In rare cases, this is the case for risky sub-portfolios. There is no benefit to being completely independent, but in most cases, it is useful to diversify the correlation between sub-combinations.It can be tested using game theory. In this way, players can use different strategies to better achieve their personal goals. In particular, we will use the results of cooperative game theory, where a group of participants can impose cooperative behavior. Competition between player alliances, not between individual players. In our case, every department is a risk taker.Have a reserve fund so that you can use it for practice. A small amount of savings. The higher the risk of the portfolio, the more reserves you should invest in. These reserves cannot be invested, so the return on investment in all business areas is much lower. It is expected that each sub-category will represent only a small part of the investment. This situation is similar toThe total cost issue is that each participant's goal is to minimize its cost, and the strategy is to accept or refuse to participate in different alliances (including all participants' alliances). The difference is that although the general cost allocation problem can have almost any structure, the risk allocation problem cannot, because the risks of different alliances are calculated by applying certain risk measures to historical data or specific locations. In this section, we first consider the important results of cooperative game theory and cost sharing theory.We then convert these results into risk allocation problems and try to understand the relationship between the selected risk metric and related risk allocation problems.

# Allocation of the atomic players

This section focuses on the distribution rules where participants are atomic (indivisible). We first consider the important findings of the cost-sharing theory. Then, we convert them into risk-sharing tasks.

Position problems or alliance games (N, c) include:

*1) Having a finite set N of n players with |N|=* *n*

*2) A cost function c:* 2*N* 🡪 *R this is defined by subset of N as c*({*Ø*}) = 0 *and c(S) is a real number of subset s of N*

The problem is set to find the allocation of the cost of the big alliance N among the participants. In our case, we use this theory to determine the size of the risk (X) between organizational units. Imagine many possible alliances: Since N has 2n possible subsets, there are 2n possible alliances (or 2n-1 possible alliances).Alliance) Alliance, if empty alliance o is omitted. You can only build an alliance that benefits all members. In other words, the fees paid by the members of the alliance are less than the fees paid by themselves. additive**.**

**Rule 1**

*For all the allocation problems it is demmed that* *if c*(*S T* ) *c*(*S* ) + *c*(*T* ) *for all subsets S and T of N with empty intersection.*

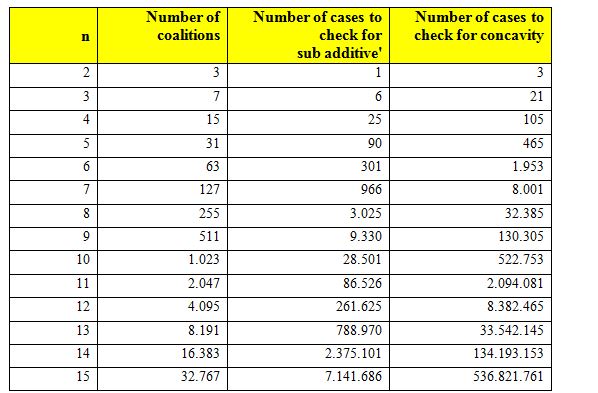
Refinement is an ideal attribute because the cost associated with collaboration must always be less than the additional cost of a single activity. The concavity is even stronger.

**Rule 2**

*We have the allocation problem deemed to concave when:*

*c*(*S T* ) + *c*(*S T* ) *c*(*S* ) + *c*(*T* ) *when all the subsets S and T of N where S*  *T .*

If the distribution problem is recessed, then participants have more incentives to form alliances. Please note that sub-addition is a special case of concave, where c(ST) = if the assignment rule is concave, then it is also a sub-addition. By confirming the gap if it is concave we check as possibilities:



Figure

When n is large, the problem becomes worse. Therefore, unless there is a computer program capable of grasping all possibilities or a computer program that explicitly creates at least one game, the sub-addition and concave test cannot be performed. At least one negative alliance or small player alliance. Obviously, the game is not subadditive, that is, it is not concave.Major League N can use allocation rules to divide between players. In fact, we require that the cost allocated to each member is the total cost of the main alliance.. In game theory, this requirement is called Pareto optimization. When deciding to join the league, each player compares their league value with their independent value. They may threaten to leave the union. The total cost exceeds your personal expenses. Such a threat may come from the player alliance, that is, if each player is assigned the same or lower level in the new sub-alliance, some players may threaten to withdraw from the alliance and form their own sub-alliance. A series of tasks with neither participants nor motivation to leave the alliance is called the core.

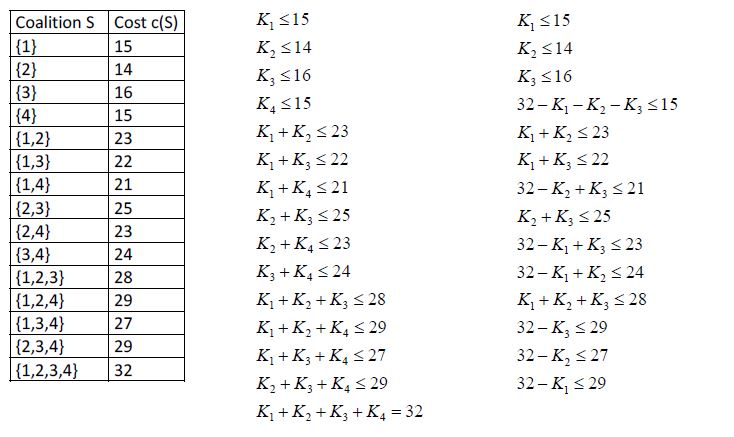
**Rule 3**

*We have that K Rn when a cost allocation is related to the allocation problem (N,c). Then*

No alliance can cut costs by abandoning Major League N. Please note that centralized interoperability means Pareto efficiency, because all risk sharing of the core business is required. No single solution can satisfy all inequalities or compact and convex subsets at the same time *Rn*.

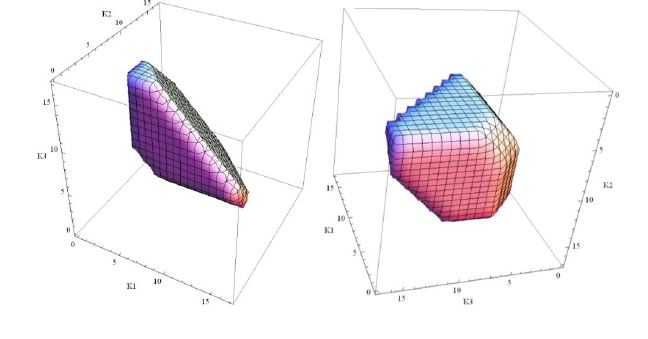
**A 4 player cost allocation problem.**

Cost allocation problem Core conditions Core conditions (alternative formulation)



Figure

The distribution problem is concave, so it is subadditive. Since the total cost of the largest coalition should reach N 32, we can restore the baseline by replacing all inequalities, as shown in the figure on the right. The empty kernel is an important attribute of the deployment problem, because the empty kernel provides a solution in which all agents are ready to collaborate.Now, we will check the attributes to ensure that the kernel is not empty.



Figure

# Alternatives of Allocation

Since these rules are given without specifying risk levels, they can be easily applied to different types of risk measures. We introduced seven different distribution rules: activity, increment, beta, cost, gap, core, Shapley and Lorenz. The first two display rules are obvious display rules for understanding and calculation.Beta method is the distribution rule in financial theory. After all, the last four distribution rules come from GameTheory. They are designed to distribute risk as fairly as possible and are therefore more complex than previous distribution rules. The "honesty" of each distribution is different, and the methods used in these distribution rules are very different. We will use cost allocation to illustrate the allocation rules.This is for illustrative purposes only, and can be used in a similar way to describe the risk of random risk dispersion problems arising from the use of a certain risk metric. These examples only show the inventory calculation for Player 1.

# Activity based method

According to the department's own risk, the overall risk is allocated to the department.

# This is a simple method which it is an advantage, but this is not considered as shortcome of this method used on the sub-units.

# Incremental method

This method allocates risk to major departments in proportion to the individual risk contribution of each business department.

# By Beta method

Having that Cov (*Xi, XN*) by writing the covariance of the subunit as *i* and the grand portfolio. So we let the to be the variance of the total portfolio. Beta unit is calculated as:

This can be allocated as:

The beta method is kept simple and takes into account the partial dependency structure. Please note that the Big Alliance N risk can be measured using only the beta method. No personal risk is required. You need to calculate the variance-covariance matrix that is not needed by other methods.It is not always certain. For example, if all portfolios are risk-free in all situations and have the same performance in all situations, all betas are 0 and the duration is is not defined.

# Lorenz method

In understanding Lorentz’s dominance, justice can also be interpreted as equality of distribution. Here, since decision-making should be centered on the game, we try to distribute the risk as evenly as possible among players. Let us define the set of Lorentz maxima as follows:

# And we should know that L(N,c) not to be unique. The definition indicates that if the pitch is the same in the core of the game, Lorentz’s distribution is the same pitch; otherwise, there is a point at the edge of the core. If there are no points in the game, Lorenz’s mission is uncertain.

# MATLAB

For example, suppose a company has two independent investment portfolios, and their returns at a certain point in the future usually average 0.05, with a standard deviation of 0.1, an average of 0.02, and a standard deviation of 0.08. Since the investment portfolio is independent, the income of a large investment portfolio can also be distributed normally. This can be calculated in Mathematica.



Figure

After allocating returns to large investment portfolios for a period of time in the future, we can adopt a certain risk measurement method, such as B Value at Risk α 5%.



Figure

Now, anticipate that the quantities invested withinside the portfolios are 100 $ and 200 $ respectively. Hence:

VaR is 22,035. This risk needs to be shared between the two investment portfolios**.** for i=1 and i=2 and the combinations of w1 and w2 that satisfy .



Figure

We know that the VaR is a linear function of gamma and obtainig the a constant to the first derivative. Found that if 0 1 and then calculation the margins which are given at one point like (100,200), which is the total capacity. We delete the values ​​w1, w2, and by doing again the calculation of w1 and w2 and having a differential of it we get the results of w1 and w2



Figure

In other words, player 1 will receive $3.72, and player 2 will receive $18.32 (two decimal places). Please note that both companies are ready to cooperate because their respective risks are 11, 45 and 22.32:



Figure

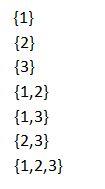
# If sub-combinations are allowed, the Aumann-Shapley method should be used, mainly because it is consistent:

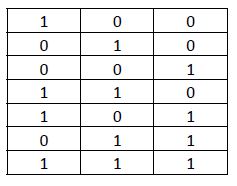
# Matlab Rules of implementation of allocation in Matlab

We discused that, some of the distribution rules used were introduced and are described in details. As the number of subunits increases, so does the number of possible alliances, and the issue of appointments becomes larger and more complex. Allocation is based on the cost/risk of multiple alliances. For example, to allocate risk using the activity method, all you need to know isRisk and the risk of large portfolio N. Even if n is relatively large, the problem can still be solved and never becomes too complicated. By the academic literature, authors often think that the main disadvantage of Shapley's method is that it is almost impossible to calculate n because of the problem. The role models are usually small, and the number of participants rarely exceeds 3 or 4. On the other hand, in practice, it is often necessary to apply distribution rules to large distribution problems with a large number of participants. Player/department/department, etc. A useful mapping problem must not only satisfy as many required theoretical attributes as possible, but also should be solvable in practice. H.It should be easy to apply matching rules to specific matching tasks. The latter is very important, because if there is no simple implementation, it is the best in theory, but in practice complex allocation rules are rarely implemented in practice. Between the theoretical nature and the simple realization of the distribution rules. In theory, the optimal allocation rules are usually very complicated (Shapley, Nucleolus), but in practice the obvious allocation rules are usually theoreticalPerformance is not ideal (incremental). With the advancement of technology today, computer programs are clearly candidates for problem solving. The allocation of risk/cost with the help of a computer program can satisfy both requirements at the same time: In theory, the optimal allocation rule can be applied as easily as the obvious allocation rule in practice: click the "Execute" button. This section focuses on implementing the above distribution rules. All allocation rules have been programmed as functions in Matlab and can be usedAssign risk/cost to any assigned task. The next section briefly introduces how Matlab works.

# Matlab program

In the case of n participants, there are 2n -1 possible alliances and associated costs/risks. The alliance can only be identified by a matrix with elements 0 and 1 (2n -1 xn). This columns are the players information, and this columns correspond to alliances. This matrix indicates whether a specific alliance contains a specific player (1 if it is, 0 otherwise). In the case of n = 3, the matrix has the following structure:





This is created as a function of S which it is equal to union n and it takes the n outputs of the union matrix S. which the input function of n is equal to 4:



Figure

In addition to using a single matrix to manage all possible alliances, this method has another important advantage: which this can be multiplied with other matrixes in order to obtain a different result. By the limitations of various alliances:



Figure

Now that we have defined the allocation problem, the obvious question is whether the problem has a non-empty kernel. However, since Matlab can program linearly, there is a simpler method. By solving it in program:

found the solution that, when -z\* is greater than the risk of Major League N, then the kernel is not empty. This is the method I used in the Matlab CE = core\_existence(S, rm) function.

# Conclusion

To this research we have shown the risk management and the cooperative of the game thoery which it is established between this two separate fields. Two different risk measures are used, namely value at risk and expected deficit. Since primer is the preferred measure of risk in the financial industry, the latter has better theoretical characteristics.The problem with VaR is the lack of subadditivity, which may be a practical problem. If the capital requirements of each business area are at its own risk, the regulator needs to be sure that the company’s total capital must also be sufficient. The combined portfolio produces trivial results, which can then lead to trivial risk capital allocations. The modeling shows that when using VaR as a risk metric, the kernel does not exist in 1-5%, depending onAlthough the expected deficit does not always satisfy the subadditivity of the discrete distribution, the absence of a core does not appear in the simulation, which makes the lack of ES subadditivity only a theoretical problem. Because only when the agreed risk measures are used, the allocation rules can be consistent.

# References

* R.J. Aumann and L.S. Shapley. Values of Non-Atomic Games. Princeton University Press, Princeton, 1974.
* J.D. Cummins. Allocation of capital in the insurance industry. Risk Management and Insurance Review, 3:7{28, 2000.
* Guttler, A. (2011). Lead–lag relationships and rating convergence among credit Rating agencies. Journal of Credit Risk, 7, 95-119.
* D. Tasche. Risk contribution and performance measurement. Technical report, Technische Universitat Munchen, 2000. <http://www-m4.mathematik.tu-muenchen.de/m4/pers/tasche/>.
* Hill, P., Brooks, R., Faff, R., (2010). Variations in sovereign credit quality assessments across rating agencies. Journal of Banking and Finance, 34, 1327-1343.
* L. Shapley. A value for n-person games. In: H.W. Kuhn and A.W. Tucker, (Eds.), Contribution to the Theory of Games, Vol. 2. Princton University Press, 1953.
* Hooper, V., Hume, T., Kim, S.J. (2008). Sovereign rating changes—do they provide new information for stock markets? Economic Systems, 32 (2), 142–166.