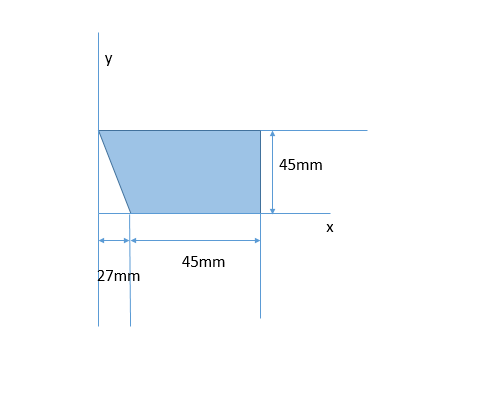
**WEEK 5: DISTRIBUTED FORCES: CENTROIDS AND CENTER OF GRAVITY**

Question 1: Determine the volume and the surface area of the solid obtained by rotating the area of Prob. 5.1 about (a) the x axis, (b) the line x= 72 mm.



**Theoretical Concept: Getting the volume, area and its centers**

**SOLUTION:**

1. **x-axis rotation**

Solving the value of the volume:

V = 2πy(area)A = 2π∑yA

=2π(63787.5 mm3)

**V = 401x103 mm3**

Solving for the Area:

Area = 2π(y2L2 + y3L3 + y4L4)

= 2π((22.5)(45) + (45)(72) + 22.5(272 + 452)

**A= 34.1x103**

1. **At line x = 72mm,**

X1 = 22.5

X2 = 36

X3 = (45+72)/2

Solving for the volume:

V = 2π(72 – X(area)A = 2π(72A -∑xA)

=2π((72mm)(2632.5mm2) – (111,172.5 mm3)

**V = 492x103 mm3**

**Solving for the area:**

A= 2πxline)L =2π∑(Xline)L

=2π(x1L1 + x3L3 + x4L4)

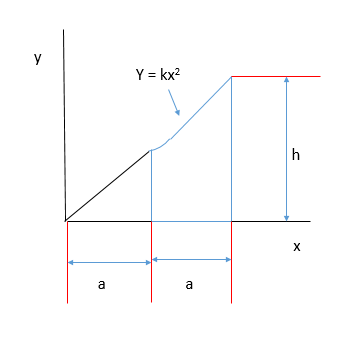
=2π((22.5)(45)+(36)(72)+(45+72)/2)(272 + 452)

**A = 41.9x103 mm2**

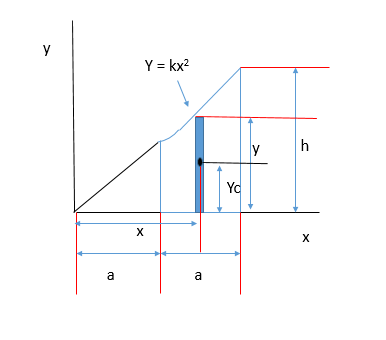
**Critical Thinking:**

**As observed from the calculated values, the higher the value of x where the rotation is done, the higher the value of volume and area we get.**

**Question 2: Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.**



**Free Body Diagram:**



h = k(2a)2

k = h/4a2

Xc = x = 2a

Yc = 0.5y = 0.5h

dA =ydx

A = = (h/4a2)

= (h/2a)(X3/3)I2aa

= (7/12)ah

= = (15/16)ha2

dx

= ½ =

= h2/32a4 (x5/5)2aa = 31/160 ah2

xA = = x(7/12ah) = 15/16 a2h

**x = 1.607a**

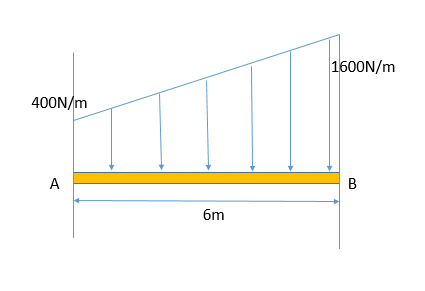
yA == y(7/12)(ah) = (31/160)ah2

**y= 0.332h**

**Critical Thinking:**

With shapes involving curves, integrating method of solving the centers is more appropriate. And the higher the value of the distance x, results to higher value of center x.

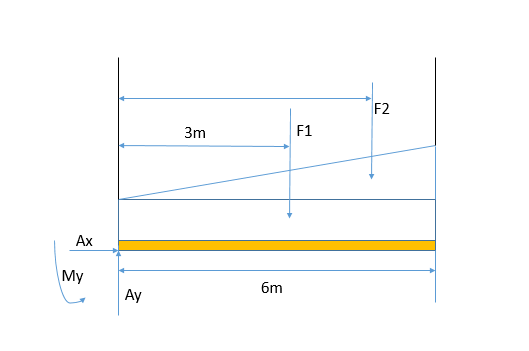
**Question 3: For the beam and loading shown, determine (a) the magnitude and location of the resultant of the distributed load, (b) the reactions at the beam supports.**



**Theoretical Concept:**

**SOLUTION:**

**Free Body Diagram:**



F1 = (400N/m)(6m) = 2400N

F2 = 1/2(1600N/m – 400N/m)(6m) = 3600N

**R = F1 + F2 = 2400N + 3600N = 6000N**

∑MA = 0: X(6000N) = (2400N)(3m) + (3600N)(1/2)(6)

**X = 3.6m**

∑Fy = 0: Ay = F1 + F2

Ay = 2400N + 3600 N

**Ay = 6000N**

∑Fx = 0: **Ax = 0 N**

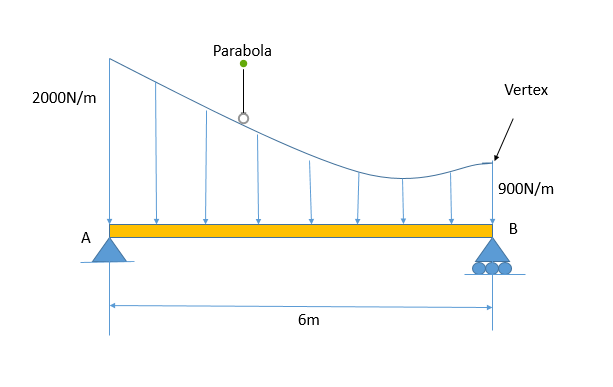
∑MA = 0: My = 6000N (3.6m)

**MA = 21.6 kN**

**Critical Thinking:**

Forces acting on a distributed load can easily be solved by cutting the distributive load into common shapes which areas can easily be determine.

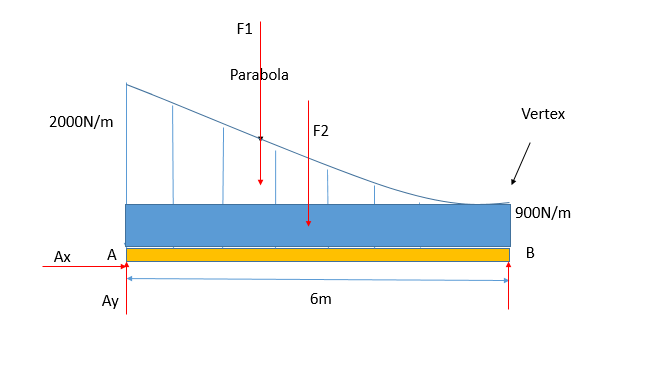
**Question 4: For the beam and loading shown, determine (a) the magnitude and location of the resultant of the distributed load, (b) the reactions at the beam supports.**



**Theoretical Concept:**

SOLUTION:

Free Body Diagram:



F1 = 1/3(2000N/m – 900N/m)(6m) = 2200N

F2 = 900N/m (6m) = 5400N

**R = F1 + F2 = 2200N + 5400N = 7600N**

X(7600N) = 2200N(1.5m) + 5400N(3m)

**X = 2.57 m**

∑MA = 0: B(6m) – 7600N(2.57m) = 0

**B= 3250 N**

∑FY = 0: A + 3250N – 7600N = 0

**A = 4350N**

**Critical thinking:**

For every parabolic distributive load, the base can be rectangular shape. Cut this part and solve the force acting on it and add it to the force acting on the parabolic area.