Q: Jones vector and matrix :-

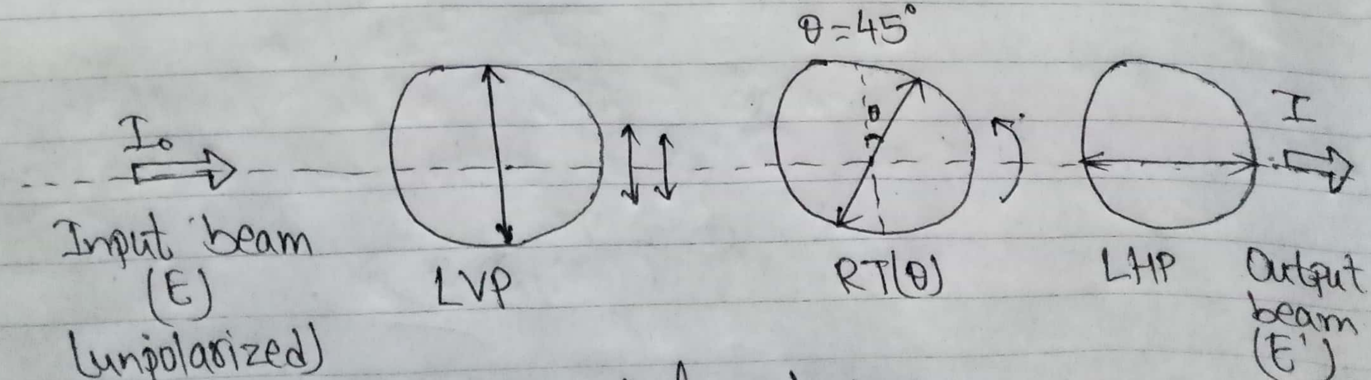
A waveplate is sandwiched between two orthogonal polarizers. The crystal axis of waveplate is aligned 45° to the input polarizer, as shown in figure below. Input light is a non-polarized light with an intensity of 10 mW/cm².

a) Please find out the intensity of the output ~~signal~~ light if the ~~input~~ phase delay of the waveplate between o-ray and e-ray is $\Gamma = \frac{\pi}{2}$.

b) Please find out the intensity of the output light if the phase delay of the waveplate between o-ray and e-ray is $\Gamma = \frac{\pi}{6}$.

c) What is the required phase delay Γ so that output light intensity is maximum?

→ Solⁿ :-



Here, LVP = Linear vertical polarizer.
RT = Waveplate or retarder.
LHP = Linear horizontal polarizer.

Also, the intensity of input beam is $I_0 = 10\ mW/cm^2$.

Let, E, E' and I be the input beam, output beam and intensity of output beam respectively.

The above problem can be solved by using Jones matrix.

We can represent the system by the eq^n,

$$E' = J.E \quad \text{——} \quad (1)$$

where,

J = the Jones matrix of whole system.

J can be calculated as,

$$J = J_{LHP} \cdot J_{RT(\theta)} \cdot J_{LVP} \quad \text{——} \quad (2)$$

We have, the Jones matrices for LHP, RT(θ) and LVP as

$$J_{LHP} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{RT(\theta)} = \begin{bmatrix} \cos\frac{\Gamma}{2} + i\sin\frac{\Gamma}{2}.\cos 2\theta & i\sin\frac{\Gamma}{2}.\sin 2\theta \\ i\sin\frac{\Gamma}{2}.\sin 2\theta & \cos\frac{\Gamma}{2} - i\sin\frac{\Gamma}{2}\cos 2\theta \end{bmatrix}$$

and,

$$J_{LVP} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Since, $\theta = 45^\circ$, we put this in $J_{RT(\theta)}$ and then eq^n (2) becomes

$$J = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos\frac{\Gamma}{2} & i\sin\frac{\Gamma}{2} \\ i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & i\sin\frac{\Gamma}{2} \\ 0 & \cos\frac{\Gamma}{2} \end{bmatrix}$$

$$\therefore \quad J = \begin{bmatrix} 0 & i\sin\frac{\Gamma}{2} \\ 0 & 0 \end{bmatrix}$$

Since the first LVP is a linear polarizer on y-axis, we can represent it using ~~Jones~~ normalized Jones vector for linear vertically polarized light as

$$E = E_0 \hat{x} = E_0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{——} \quad (3)$$

We know,

$$I_0 = E_0^2$$

or, $$E_0 = \sqrt{I_0}$$

$$\therefore \quad E_0 = \sqrt{10} = 3.162$$

Substituting this value in eq^n (3), we get

$$E = 3.162 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Now, using eq^n (1), we can write,

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$$E' = J \cdot E$$

$$= \begin{bmatrix} 0 & i\sin\frac{\Gamma}{2} \\ 0 & 0 \end{bmatrix} \cdot 3.162 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

or, $$E' = 3.162 \begin{bmatrix} i\sin\frac{\Gamma}{2} \\ 0 \end{bmatrix}$$

or, $$E' = i \cdot 3.162 \begin{bmatrix} \sin\frac{\Gamma}{2} \\ 0 \end{bmatrix}$$

The factor i in above eq^n is unimodular and can be suppressed. It is common, so, we may write

$$E' = 3.162 \begin{bmatrix} \sin\frac{\Gamma}{2} \\ 0 \end{bmatrix}$$

The complex transponse of E' is

$$(E')^* = 3.162 \left[\sin\frac{\Gamma}{2} \quad 0 \right]$$

Now, the output intensity can be calculated as,

$$I = (E')^* \cdot E'$$
$$= (3.162)^2 \begin{bmatrix} \sin\frac{\Gamma}{2} & 0 \end{bmatrix} \begin{bmatrix} \sin\frac{\Gamma}{2} \\ 0 \end{bmatrix}$$

$$\therefore \quad I = 10\left(\sin^2\frac{\Gamma}{2}\right) \quad \text{———} \quad (4)$$

As we can clearly see, the output intensity depends on the phase delay of waveplate between o-ray and e-ray i.e. Γ.

a) For $\Gamma = \pi/2$,
$$I = 10 \sin^2\frac{\pi}{4} = 5 \text{ mW/cm}^2$$
Ans.

b) For $\Gamma = \pi/6$,
$$I = 10 \sin^2\left(\frac{\pi}{12}\right) = 0.6699 \text{ mW/cm}^2$$
Ans.

c) From eq^n (4), we see that the value of intensity, I varies with $\sin^2\frac{\Gamma}{2}$. The maximum light intensity is obtained when,
$$\sin^2\frac{\Gamma}{2} = 1$$
$$\text{or,} \quad \sin\frac{\Gamma}{2} = 1 \quad \text{[taking only positive value]}$$
$$\text{or,} \quad \sin\frac{\Gamma}{2} = \sin\frac{\pi}{2}$$

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Or, $\frac{\Gamma}{2} = \frac{\pi}{2}$

$\therefore \boxed{\Gamma = \pi = \Gamma_{max}}$

Hence, the intensity of output light is maximum when $\Gamma_{max} = \pi$.

Ans.