DERIVATIVES ASSIGNMENT HELP

Problem 1

A bank offers a corporate client a choice between borrowing cash at 11% p.a. and borrowing gold at 2% p.a. (If gold is borrowed, interest must be repaid in gold. Thus, 100 ounces of gold borrowed today, would require 102 ounces to be repaid in 1 year). The risk free interest rate is at 9.25% p.a., and storage costs are 0.5% p.a.

Discuss whether the interest rate on gold loan is too high or too low in relation to the rate of interest on the cash loan. The interest rates on the two loans are expressed with annual compounding. The risk free interest rate and the storage costs are expressed with continuous compounding.

Solution to Problem 1

The investor has two options:
Option 1: Borrow cash.
Option 2: Borrow gold.

To check whether the interest rate on gold loan is too high or too low in relation to the rate of interest on the cash loan let us consider 'r' as the interest rate charged on gold that would make the two loans equivalent.

Option 1:
Borrow money at 11% p.a., buy gold with the funds obtained and pay 0.5% storage costs for gold. At the same time, enter into a forward contract to sell gold at loan maturity. At loan maturity, sell gold at the forward price.

Let us denote the current spot price of gold as S.

Then the forward price is $F = S*e^{0.0925+0.005} = Se^{0.098}$

Assume that the investor borrows \$100. With this funds he will buy 100/S ounces of gold, and sell them for $(100/S)* Se^{0.098} \approx 100*e^{0.098} \approx \$110.30$

But the loan repayment is \$100 * (1.11) = \$111, hence, that the investor is losing \$111 - $\$100*e^{0.098} \approx$ \$0.70 on the transaction, paid at loan maturity.

Option 2:
Borrow 100/S ounces of gold, and repay the loan with (100/S)(1+r) ounces of gold.

Thus the investor loses (100/S)*(1+r) – 100/S = 100r/S ounces of gold in this transaction, paid at loan maturity.

As the arbitrage free forward price of gold is $Se^{0.098}$, the dollar cost of that loss is $(100r/S)*(Se^{0.098}) = 100r\ e^{0.098}$

For the two transactions to be equal, we must have

$\$111 - \$100*e^{0.098} = 100r\ e^{0.098}$
Or,

$$\text{Or, } r = \frac{\$111 - \$100*e^{0.098}}{\$100\ e^{0.098}}$$

Or, r = 0.7%

Thus the gold rate is less than the interest rate on the cash loan. It is advantageous for the investor to borrow gold.

Problem 2

A fund manager has portfolio worth $50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next two months and plans to use 3-month future contracts on the S&P 500 to hedge the risk. The current level of the index is 1,250, one contract is on 250 times the index, the risk free rate is 6% p.a., and the dividend yield on the index is 3% p.a. The current 3-moth future price is 1259.

a) What position should the fund manager take to hedge all exposure to the market over the next two months?

b) Calculate the effect of your strategy on the fund managers' return if the index in 2 months is 1,000, 1,100, 1,200, 1,300, 1,400. Assume that the 1-month future price is 0.25% higher than the index level at this time.

Solution to Problem 2(a)

The Portfolio Hedging Program is a managed hedge program which uses stock index futures and/or options to transfer price risk associated with long equity positions. Instead of selling individuals stocks investors can create a substitute sale through a short position in index futures and/or options. To participate in such a program, determinations would need to be made as to how an investor's stock portfolio correlates to a particular index. Once the "beta", correlation of an individual stock or group of stocks to an index, is calculated, the proper hedge ratio can be determined as under:

$$\text{Number of contracts to hedge} = \frac{\text{Weighted beta of portfolio}(\beta) \text{ x \$ Value of portfolio}(P)}{\text{\$ Value of index}}$$

$$\text{No. of contracts} = \frac{0.87 \text{ x } \$50{,}000{,}000}{\$1{,}250 \text{ x } 250} = 139.2$$

Rounding to the nearest whole number, the investor should short 139 contracts to eliminate exposure to the market.

Solution to Problem 2(b)

Case I:

If the index in two month is 1,000 the 1-month future price which is 0.25% higher than the index level at this time can be calculated as under:

Futures price = 1,000 * 1.0025 = 1,002.50.
Thus the gain on the short futures position = (1,259 − 1,002.50) × 250 × 139 = $8,913,375.

The return on the index for the index value of 1000 can be calculated as follows:

Return in the form of Dividend (3/6)	0.50%
Return in the form of Capital Gains (1000-1250)/1250	-20.00%
Total Returns	-19.50%
Risk free rate for two months (6%*2/12)	1.00%
Returns in excess of risk free rate	-20.50%
Expected Return on portfolio in excess of risk free rate from CAPM (0.87*-20.5%)	-17.835%
Portfolio Return	-16.835%

The loss on the portfolio is 0.16835 ×$50,000,000 or $8,417,500. When this is combined with the gain on the futures the total gain is $495,875. The calculations for the return on index at other index levels are the same.

The following table shows that the strategy has the effect of locking in a gain close to $490,000 at any level of the index.

Index in two months	1,000	1,100	1,200	1,300	1,400
Futures Price	1,002.50	1,102.75	1,203.00	1,303.25	1,403.50
Gain on futures($)	8,913,375	5,429,688	1,946,000	(1,537,688)	(5,021,375)
Index Return	-19.5	-11.5	-3.5	4.5	12.5
Excess Index return	-20.5	-12.5	-4.5	3.5	11.5
Excess Portfolio Return	-17.835	-10.875	-3.915	3.045	10.005
Portfolio return	-16.835	-9.875	-2.915	4.045	11.005
Portfolio Gain($)	(8,417,500)	(4,937,500)	(1,457,500)	2,022,500	5,502,500
Total Gain	495,875	492,188	488,500	484,813	481,125

Problem 3

The author's Web page (www.rotman.utoronto.ca/'~hull/data) contains daily closing prices for crude oil and gold futures contracts. (Both contracts are traded on NYMEX.)You are required to download the data and answer the following:

(a) How high do the maintenance margin levels for oil and gold have to be set so that there is a 1 % chance that an investor with a balance slightly above the maintenance margin level on a particular day has a negative balance 2 days later? How high dothey have to be for a 0.1% chance? Assume daily price changes are normally distributed with mean zero. Explain why NYMEX might be interested in this calculation.

(b) Imagine an investor who starts with a long position in the oil contract at the beginning of the period covered by the data and keeps the contract for the whole of the period of time covered by the data. Margin balances in excess of the initial margin are withdrawn. Use the maintenance margin you calculated in part (a) for a 1% risk level and assume that the maintenance margin is 75% of the initial margin. Calculate the number of margin calls and the number of times the investor has a negative margin balance. Assume that all margin calls are met in your calculations. Repeat the calculations for an investor who starts with a short position in the gold contract.

Solution to Problem 3 (a)

CRUDE OIL

Standard deviation of daily changes in price of the crude oil futures contract = $0.31 per barrel or $310 per contract.

For a 1% risk
The z statistic for a level of significance of 1% is 2.33.
For a 1% chance that an investor with a balance slightly above the maintenance margin level on a particular day has a negative balance 2 days later, the required maintenance margin level for crude = $310*√2*2.33 =$1,021.

Similarly for a 0.1% risk, the required maintenance margin =$310*√2*3.09 = $1,355

GOLD

Standard deviation of daily changes in price of gold futures contract = $2.77 per ounce or $277 per contract.

For a 1% risk
The z statistic for a level of significance of 1% is 2.33

For a 1% chance that an investor with a balance slightly above the maintenance margin level on a particular day has a negative balance 2 days later, the required maintenance margin level for gold =$277*√2*2.33 = $912.

For a 0.1% risk, the required maintenance margin = $277*√2*3.09 = $1,210.

Solution to Problem 3 (b)

CRUDE

The initial margin is set at $1362. There are 151 margin calls and 7 times out of 1201 days where the investor is tempted to walk away. With the risk level of 0.1%, there are 3 occasions when the oil investor might walk away

GOLD

The initial margin is set at $1217. There are 93 margin calls and 9 times out of 823 days where the investor is tempted to walk away. With the risk level of 0.1% there are 6 occasions when the gold investor might walk away.

The results above suggest that the extreme movements occur more often than the normal distribution would suggest. For reference, an excel workbook showing the calculation for crude oil with a 1% risk has been attached. The formulas used are shown as under:

Suppose that the initial margin is in cell Q1 and the maintenance margin in cell Q2. We calculate the change in the oil futures price in column D of the spreadsheet as today's price – previous days price, and the margin balance at days close column E.
Consider cell E7. This is updated with an instruction of the form
=IF(E6<Q2,Q1,IF(E6+D7*1000>Q1,Q1,E6+D7*1000)
Putting 1 in cell F7 if there has been a margin call and zero otherwise requires an instruction of the form:
=IF(E7<Q2,1,0)
Putting 1 in cell G7 if there has been an incentive to walk away and zero otherwise requires an instruction of the form:
=IF(E6+D7*1000<0,1,0)

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